Total No. of printed pages = 10

Sc-102/Maths-I/1st Sem(New)/Com/2017/N

MATHEMATICS – I

(New Course)

Full Marks – 70

Time – Three hours

The figures in the margin indicate full marks for the questions.

PART – A

1. Choose the correct answer : $1\times10=10$

(a) Square root of $3 + 4i$ is

(i) $\pm(4 + i)$ (ii) $\pm(1 + i)$

(iii) $\pm(2 - i)$ (iv) $\pm(2 + i)$

(b) Modulus of $\frac{2-i}{3-4i}$ is

(i) $\frac{1}{5}$ (ii) $\frac{1}{\sqrt{5}}$

(iii) $\sqrt{3}$ (iv) 5

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(c) Value of $\log_2\log_2\log_3 81$ is

(i) 2 (ii) 3

(iii) 1 (iv) None of the above

(d) Value of ω^{105} is

(i) -1 (ii) 1

(iii) ω (iv) $-\omega$

(e) arg $(4 - i4)$ is

(i) π (ii) $\pi/2$

(iii) $\pi/4$ (iv) $-\pi/4$

(f) Sum of first 24 terms in $-9 - 1 + 7 + \ldots\ldots$ is

(i) 1992 (ii) 1662

(iii) 4620 (iv) None of the above

(g) 6th term of 2, 8, 32, ... is

(i) 563 (ii) 2408

(iii) 4902 (iv) 2048

(h) Number of ways that the letters of the word DEER be arranged is

(i) 10 (ii) 8

(iii) 12 (iv) 13

(i) Expansion of $(1 + x)^{-1}$ is

(i) $1 - x + x^2 - x^3 + x^4 - \ldots\ldots$ to infinity

(ii) $1 + x + x^2 + x^3 + x^4 + \ldots\ldots$ to infinity

(iii) $1 - x + x^2 - x^3 + x^4 - \ldots\ldots + x^n$

(iv) $1 + x + x^2 + x^3 + x^4 + \ldots\ldots + x^n$

(j) Cofactor of a_{23} in $\begin{vmatrix} 2 & -1 & 0 \\ 1 & -2 & 1 \\ 4 & 3 & -1 \end{vmatrix}$ is

(i) $\begin{vmatrix} 2 & -1 \\ 1 & -2 \end{vmatrix}$ (ii) $-\begin{vmatrix} 2 & -1 \\ 1 & -2 \end{vmatrix}$

(iii) $-\begin{vmatrix} 2 & -1 \\ 4 & 3 \end{vmatrix}$ (iv) $\begin{vmatrix} 2 & 0 \\ 4 & -1 \end{vmatrix}$

2. Choose the correct statement : $1 \times 5 = 5$

(a) (i) $\sin^2 x + \cos^2 x = 1$

(ii) $\sec^2 x + \operatorname{cosec}^2 x = 2$

(iii) $\cos^2 x - \cot^2 x = -1$

(b) (i) $-1 < \cos x < 1$ (ii) $\cos x \leq 1$

(iii) $-1 \leq \sin x \leq 1$

(c) (i) $1 + \sin A = \left(\sin\frac{A}{2} - \cos\frac{A}{2}\right)^2$

(ii) $\cos A = 2\sin^2\frac{A}{2} + 1$

(iii) $1 - \cos A = 2\sin^2\frac{A}{2}$

(d) (i) $\tan(a+b+c) =$

$$\frac{\tan a + \tan b + \tan c - \tan a \tan b \tan c}{1 - \tan a \tan b - \tan b \tan c - \tan c \tan a}$$

(ii) $\tan(a+b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$

(iii) $\tan(45^\circ + A)\tan(45^\circ - A) = -1$

(e) (i) $\frac{a}{\cos A} = \frac{b}{\cos B} = \frac{c}{\cos C}$

(ii) $\cos\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$

(iii) $a = b\cos C + c\cos B$

3. Find the correct answer : $1 \times 5 = 5$

(a) The cost of digging a pit of size $4 \times 5 \times 4$ at the rate of Rs. 50 is

(i) Rs. 4,000 (ii) Rs. 2,000

(iii) Rs. 3,500 (iv) Rs. 3,650

(b) The length of the longest rod that can be kept in a box of size $3 \times 12 \times 4$ is

(i) 7.9 (ii) 8.2

(iii) 12.5 (iv) 13

(c) The volume of a sphere of radius 6 is

(i) 287π (ii) 346π

(iii) 410π (iv) 288π

(d) The base radius of a cone is 7. If the height of the pyramid is 24 cm, its lateral surface is

(i) 175π (ii) 174

(iii) 238π (iv) 188π

(e) The height of a cylinder is 6 cm and the ratio to its volume to the lateral surface area is 2 : 1. The radius is

(i) 4.5 (ii) 3

(iii) 4 (iv) 2.5

4. Choose the correct statement : $1 \times 5 = 5$

(a) (i) Distance between (4, 1) and (3, 0) is $\sqrt{2}$.

(ii) Gradient of the line joining (4, 1) and (3, 0) is 2.

(iii) Origin is a point on the line joining (4, 1) and (3, 0).

(b) (i) Two lines are parallel if coefficient of y in the two equations are same.

(ii) Two lines are mutually perpendicular if product of their gradient is 1.

(iii) If constant term of an equation of a straight line is 0, then the line passes through the origin.

(c) (i) The x intercept of $2x - 3y + 1 = 0$ is 2.

(ii) The y intercept of $3x - y + 6 = 0$ is 6.

(iii) $2x - 3y + 1 = 0$ and $5x - 3y + 5 = 0$ are parallel lines.

(d) (i) The gradient form of the equation $2x + y = 4$ is $y = -2x + 4$.

(ii) The gradient form of the equation $5x + 2y = 1$ is $y = -5x + 1$.

(iii) The intercept form of the equation $x + y = 6$ is $\frac{x}{6} - \frac{y}{6} = 1$.

(e) (i) Equation of the line passing through (2, 1) and (4, 6) is $5x - 2y = 8$.

(ii) Equation of the line passing through (0, 0) and (1, 2) is $2x - y = 7$.

(iii) Equation of the line passing through (–3, 1) and (3, 3) is $x + 2y + 6 = 0$.

Part – B

5. Answer any *five* questions : $2 \times 5 = 10$

(i) Evaluate $\log_2 \log_3 \log_2 512$.

(ii) If $x = 1 - i$, find the value of $x^2 - 2x + 2$.

(iii) If ${}^nP_3 = 336$, find ${}^nC_3$.

(iv) Determine the value of k if $7k + 3$, $4k - 5$, $2k + 10$ are in AP.

(v) Find 8th term in $\left(1 + \frac{1}{x}\right)^{17}$.

(vi) Apply De Mov's theorem to find the value $(1 + i)^2$.

(vii) In how many ways can the letters of the word MULTIPLE be arranged without changing the order of the vowels in the word ?

(viii) How many chords can be drawn through 11 points on a circle ?

6. Find the value of $\left(\sqrt{2}+1\right)^5-\left(\sqrt{2}-1\right)^5$. 3

7. Answer any *two* questions : $4\times2=8$

(i) If ω is an imaginary cube root of unity, prove that $\frac{1}{1+2\omega}+\frac{1}{2+\omega}-\frac{1}{1+\omega}=0$.

(ii) Insert 5 GMs between 576 and 9.

(iii) Prove that $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

(iv) Resolve into simple fraction : $\frac{x^2}{(x+1)^2(x+2)}$

8. Prove that (any *four*) : $2\times4=8$

(i) $\sin^2 48^\circ + \sin^2 42^\circ = 1$

(ii) $\tan 53^\circ = \dfrac{\cos 8^\circ + \sin 8^\circ}{\cos 8^\circ - \sin 8^\circ}$

(iii) $\tan\left(\dfrac{\pi}{4} + \dfrac{\theta}{2}\right) = \sec\theta + \tan\theta$

(iv) $\cos^4\theta - \sin^4\theta = \cos 2\theta$

(v) $\cos 130^\circ + \cos 110^\circ + \cos 10^\circ = 0$

(vi) $\dfrac{\cos\theta + \cos\phi}{\sin\theta - \sin\phi} = \dfrac{\sin\theta + \sin\phi}{\cos\phi - \cos\theta}$

(vii) $\dfrac{\sin(B-C)}{\cos B \cos C} = \tan B - \tan C$

9. Answer any *two* questions : $3\times2=6$

(i) If $A + B + C = \pi$, prove that $\sin^2A + \sin^2B + \sin^2C = 2 + 2\cos A \cos B \cos C$

(ii) Prove that $\tan^{-1}\dfrac{5}{12} = \sin^{-1}\dfrac{5}{13} = \cos^{-1}\dfrac{12}{13}$

(iii) For the triangle ABC, prove that $\tan\dfrac{A-B}{2} = \dfrac{a-b}{a+b}\cot\dfrac{C}{2}$

10. A river is 32m wide. The depth d in meters at a distance x m from one bank is given by the following table :

x :	0	4	8	12	16	20	24	28	32
d :	0	10	20	25	30	41	44	26	10

Find the approximate cross-section of the river. 3

11. Answer any *two* questions : $2\times2=4$

(i) Show that $(-1, -1)$, $(1, 1)$ and $(-\sqrt{3}, \sqrt{3})$ are the vertices of an equilateral triangle.

(ii) Divide the line joining $(-1, 1)$ and $(6, 8)$ internally in the ratio $2 : 1$.

(iii) Find locus of a point moving at a constant distance 3 from $(4, 1)$.

(iv) Find the length of perpendicular from $(0, 0)$ to $x - 5y - 9 = 0$.

12. Find the angle between the lines $x + 2y - 1 = 0$ and $6x + 5y - 3 = 0$. 3

Or

Show that the points (2, 3) (3, 5) and (6, 11) are collinear.